



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact [support@jstor.org](mailto:support@jstor.org).

NOTE BY PROF. W. W. JOHNSON.— Problem 85, in ANALYST for September, is identical with No. 85 of the Exercises in Chauvenet's Geometry, the statement being made in the form which Prof. Scheffer gives in the November No. Prof. Scheffer is undoubtedly right in stating that the problem cannot be solved by the rule and compass only. The following will show that it has theoretically six solutions, and in some cases they are all real solutions.

Assume the given point as origin and the line joining it with the centre of one of the circles as initial line, the polar equation of that circle is

$$r^2 - 2ar \cos \theta + a^2 = b^2,$$

or 
$$r = a \cos \theta \pm \sqrt{(b^2 - a^2 \sin^2 \theta)}.$$

Construct a curve by increasing and diminishing each radius vector by  $d$ , the length of the fourth side of the quadrilateral or line to be placed between the circumferences. The equation to this curve is therefore

$$r = a \cos \theta \pm d \pm \sqrt{(b^2 - a^2 \sin^2 \theta)},$$

or 
$$r^2 - 2ar \cos \theta \mp 2d(r - a \cos \theta) + a^2 + d^2 - b^2 = 0,$$

or, partially introducing rectangular coordinates,

$$x^2 + y^2 - 2ax + a^2 + d^2 - b^2 = \pm 2d(r - a \cos \theta).$$

Multiplying both members by  $r$  and squaring we have

$$(x^2 + y^2)(x^2 + y^2 - 2ax + a^2 + d^2 - b^2)^2 = 4d^2(x^2 + y^2 - ax)^2. \quad (1)$$

Now whenever this locus is cut by the other circle we have a solution of the problem. The equation of this last circle will be of the form

$$x^2 + y^2 + Ax + By + C = 0. \quad (2)$$

Substituting the value of  $x^2 + y^2$  from (2) into (1), (namely the linear expression,  $-Ax - By - C$ ) we have an equation of the third degree; which cubic combined with equation (2) will give theoretically six values of  $x$  and  $y$ . Constructing the curve (1) in the case when the distance of the pole from the circumference of the first circle is less than  $d$ , it is evident that the circle (2) may cut (1) in six real points, hence there may be six real solutions. The position of the given circles and the given point in this case is as follows — The circles being of unequal radii intersect, and the given point is within the smaller of the crescent-shaped figures formed. Let a straight line pass through the given point and revolve about it, the intercepts made on this line by each of the crescent-shaped and the lens-shaped figures will vary, and each may evidently twice become equal to the line to be inscribed provided the latter is within the proper limits.